

# IB Mathematical Studies SL

## Summer Review Packet – Year 2

**Instructions: Complete each week's problem set. Show your work. Do not forget to either round to three significant figures or provide the exact answer. Include units where appropriate. This will count as a summative grade. It will be graded for accuracy.**

Name \_\_\_\_\_

Date \_\_\_\_\_

**Teacher: Ms. Billue**

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# Week 1:

1a. [2 marks]

A group of 20 students travelled to a gymnastics tournament together. Their ages, in years, are given in the following table.

<b>Age (years)</b>	14	15	16	17	18	19	20	22
<b>Frequency</b>	1	2	7	1	4	1	1	3

For the students in this group find the mean age;

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1b. [1 mark]

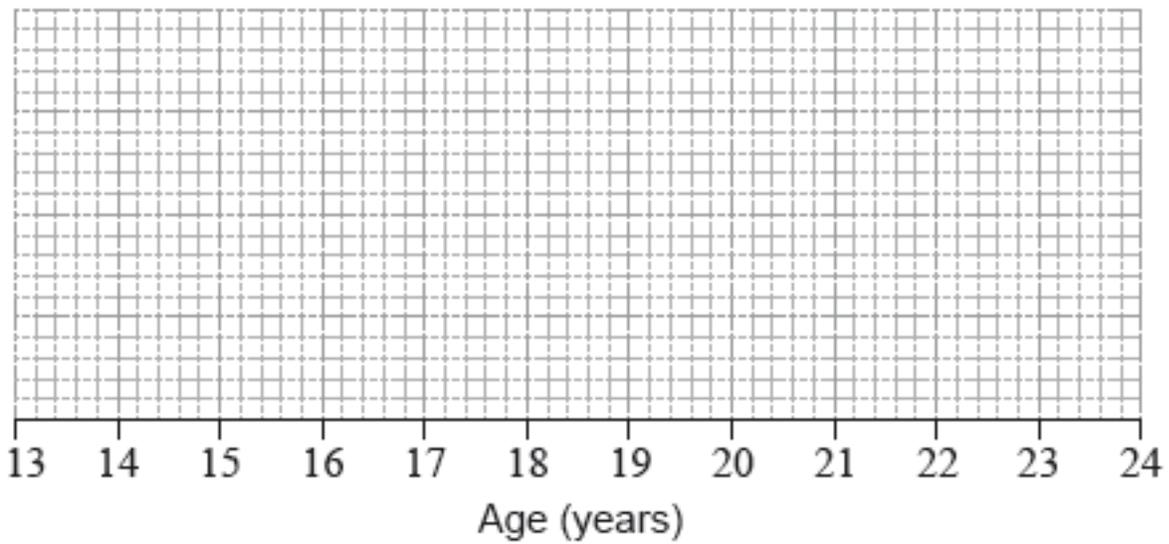
For the students in this group write down the median age.

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1c. [3 marks]

The lower quartile of the ages is 16 and the upper quartile is 18.5.

Draw a box-and-whisker diagram, for these students' ages, on the following grid.



2a. [2 marks]

The coordinates of point A are  $(6, -7)$  and the coordinates of point B are  $(-6, 2)$ . Point M is the midpoint of AB.

Find the coordinates of M.

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2b. [2 marks]

$L_1$  is the line through A and B.

Find the gradient of  $L_1$ .

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**2c.** [1 mark]

The line  $L_2$  is perpendicular to  $L_1$  and passes through M.

Write down the gradient of  $L_2$ .

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**2d.** [1 mark]

Write down, in the form  $y = mx + c$ , the equation of  $L_2$ .

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**3a.** [3 marks]

The speed of light is **300 000** kilometres per second. The average distance from the Sun to the Earth is 149.6 million km.

Calculate the time, **in minutes**, it takes for light from the Sun to reach the Earth.

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**3b.** [3 marks]

A light-year is the distance light travels in one year and is equal to **9 467 280** million km. Polaris is a bright star, visible from the Northern Hemisphere. The distance from the Earth to Polaris is 323 light-years.

Find the distance from the Earth to Polaris in millions of km. Give your answer in the form  $a \times 10^k$  with  $1 \leq a < 10$  and  $k \in \mathbb{Z}$ .

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## Week 2:

4a. [2 marks]

**In this question, give all answers correct to 2 decimal places.**

Jose travelled from Buenos Aires to Sydney. He used Argentine pesos, ARS, to buy 350 Australian dollars, AUD, at a bank. The exchange rate was  $1 \text{ ARS} = 0.1559 \text{ AUD}$ .

Use this exchange rate to calculate the amount of ARS that is equal to 350 AUD.

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4b. [2 marks]

The bank charged Jose a commission of 2%.

Calculate the **total** amount of ARS Jose paid to get 350 AUD.

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**4c.** [2 marks]

Jose used his credit card to pay his hotel bill in Sydney. The bill was 585 AUD. The value the credit card company charged for this payment was 4228.38 ARS. The exchange rate used by the credit card company was 1 AUD =  $x$  ARS. No commission was charged.

Find the value of  $x$ .

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**5a.** [2 marks]

Juan buys a bicycle in a sale. He gets a discount of 30% off the original price and pays 560 US dollars (USD).

Calculate the original price of the bicycle.

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**5b.** [4 marks]

To buy the bicycle, Juan takes a loan of 560 USD for 6 months at a nominal annual interest rate of 75%, **compounded monthly**. Juan believes that the total amount he will pay will be less than the original price of the bicycle.

Calculate the difference between the original price of the bicycle and the total amount Juan will pay.

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**6a.** [1 mark]

Rosa joins a club to prepare to run a marathon. During the first training session Rosa runs a distance of 3000 metres. Each training session she increases the distance she runs by 400 metres.

Write down the distance Rosa runs in the third training session;

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6b. [2 marks]

Write down the distance Rosa runs in the  $n$ th training session.

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6c. [2 marks]

A marathon is 42.195 kilometres.

In the  $k$ th training session Rosa will run further than a marathon for the first time.

Find the value of  $k$ .

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6d. [4 marks]

Calculate the total distance, in **kilometres**, Rosa runs in the first 50 training sessions.

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6e. [3 marks]

Carlos joins the club to lose weight. He runs 7500 metres during the first month. The distance he runs increases by 20% each **month**.

Find the distance Carlos runs in the fifth month of training.

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6f. [3 marks]

Calculate the total distance Carlos runs in the first year.

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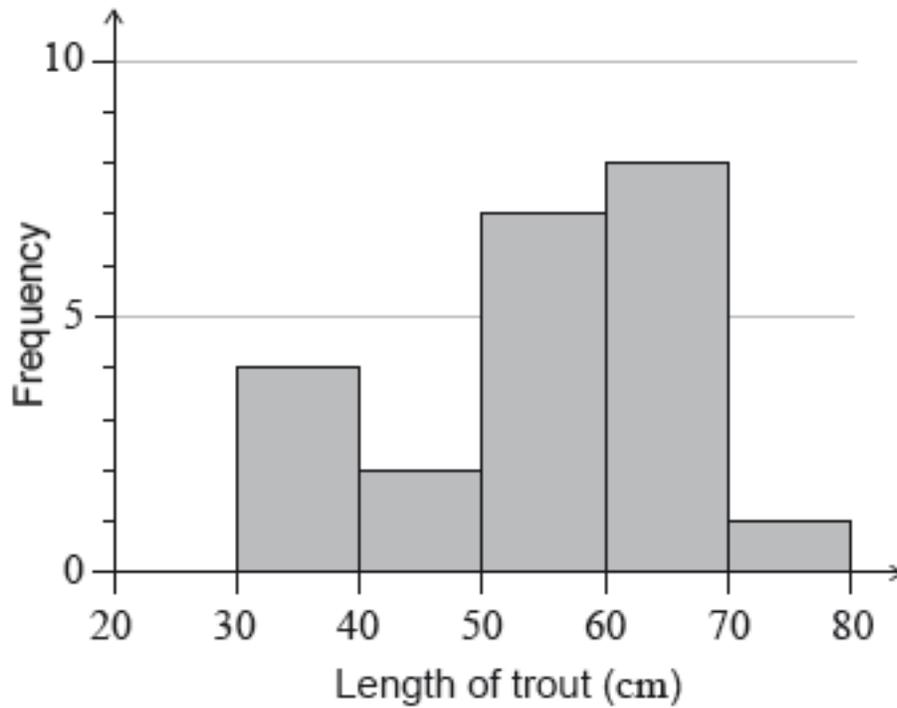
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## Week 3:

7a. [2 marks]

The lengths of trout in a fisherman's catch were recorded over one month, and are represented in the following histogram.



Complete the following table.

Length of trout	Frequency
20 cm < trout length ≤ 30 cm	0
30 cm < trout length ≤ 40 cm	
40 cm < trout length ≤ 50 cm	
50 cm < trout length ≤ 60 cm	
60 cm < trout length ≤ 70 cm	
70 cm < trout length ≤ 80 cm	1

7b. [1 mark]

State whether **length of trout** is a continuous or discrete variable.

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7c. [1 mark]

Write down the modal class.

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7d. [2 marks]

Any trout with length 40 cm or less is returned to the lake.

Calculate the percentage of the fisherman's catch that is returned to the lake.

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**8a.** [2 marks]

The first three terms of a geometric sequence are  $u_1 = 486$ ,  $u_2 = 162$ ,  $u_3 = 54$ .

Find the value of  $r$ , the common ratio of the sequence.

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**8b.** [2 marks]

Find the value of  $n$  for which  $u_n = 2$ .

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**8c.** [2 marks]

Find the sum of the first 30 terms of the sequence.

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9a. [1 mark]

The equation of line  $L_1$  is  $y = -\frac{2}{3}x - 2$ .

Write down the gradient of  $L_1$ .

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9b. [2 marks]

Point P lies on  $L_1$  and has  $x$ -coordinate  $-6$ .

Find the  $y$ -coordinate of P.

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9c. [3 marks]

The line  $L_2$  is perpendicular to  $L_1$  and intersects  $L_1$  when  $x = -6$ .

Determine the equation of  $L_2$ . Give your answer in the form  $ax + by + d = 0$ , where  $a$ ,  $b$  and  $d$  are integers.

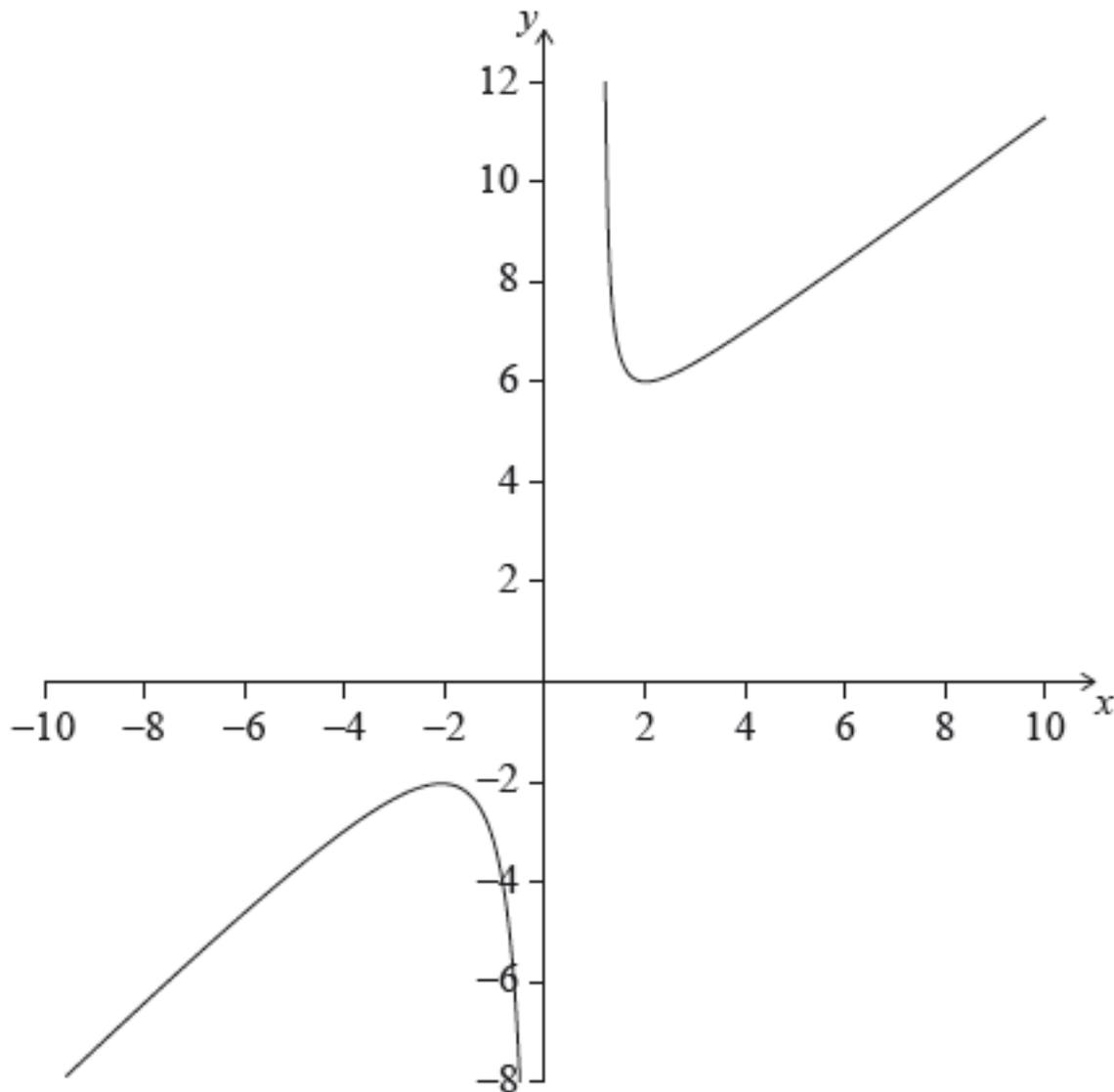
A rectangular box with a solid black border. Inside the box, there are six horizontal dotted lines, evenly spaced, intended for handwriting practice.

## Week 4:

10a. [2 marks]

The function  $f$  is of the form  $f(x) = ax + b + \frac{c}{x}$ , where  $a$ ,  $b$  and  $c$  are positive integers.

Part of the graph of  $y = f(x)$  is shown on the axes below. The graph of the function has its local maximum at  $(-2, -2)$  and its local minimum at  $(2, 6)$ .



Write down the domain of the function.

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**10b.** [1 mark]

Draw the line  $y = -6$  on the axes.

**10c.** [1 mark]

Write down the number of solutions to  $f(x) = -6$ .

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**10d.** [2 marks]

Find the range of values of  $k$  for which  $f(x) = k$  has no solution.

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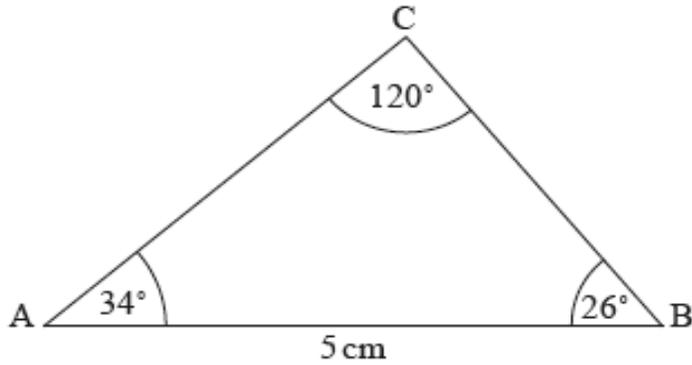
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11a. [3 marks]

A triangular postage stamp, ABC, is shown in the diagram below, such that  $AB = 5 \text{ cm}$ ,  $\hat{BAC} = 34^\circ$ ,  $\hat{ABC} = 26^\circ$  and  $\hat{ACB} = 120^\circ$ .

diagram not to scale



Find the length of BC.

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11b. [3 marks]

Find the area of the postage stamp.

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**12a.** [2 marks]

In the Canadian city of Ottawa:

97% of the population speak English,

38% of the population speak French,

36% of the population speak both English and French.

Calculate the percentage of the population of Ottawa that speak English but not French.

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**12b.** [2 marks]

The total population of Ottawa is 985 000.

Calculate the number of people in Ottawa that speak both English and French.

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12c. [2 marks]

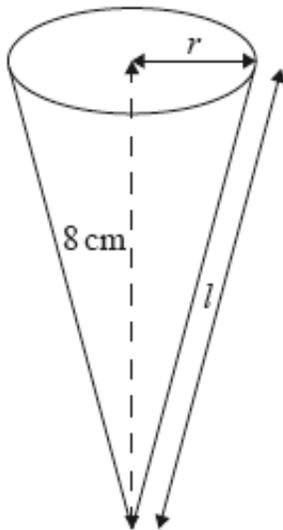
Write down your answer to part (b) in the form  $a \times 10^k$  where  $1 \leq a < 10$  and  $k \in \mathbb{Z}$ .

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# Week 5:

13a. [2 marks]

A type of candy is packaged in a right circular cone that has volume  $100 \text{ cm}^3$  and vertical height 8 cm.



Find the radius,  $r$ , of the circular base of the cone.

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13b. [2 marks]

Find the slant height,  $l$ , of the cone.

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13c. [2 marks]

Find the curved surface area of the cone.

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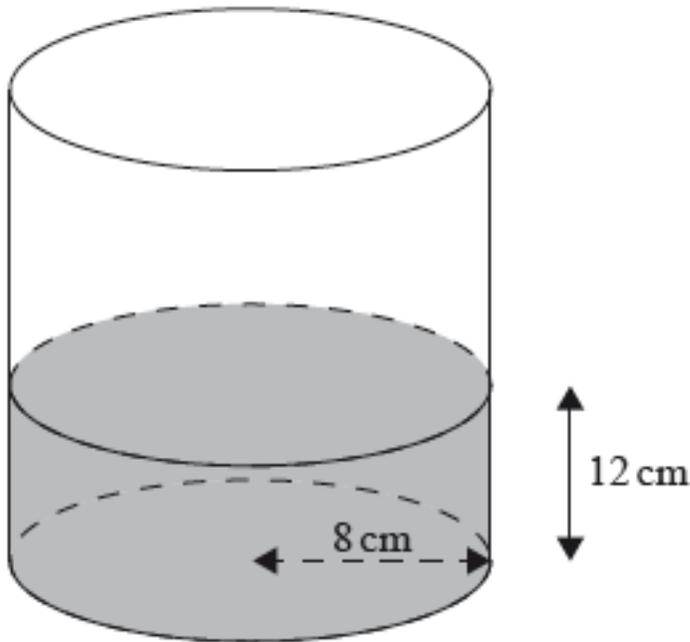
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14a. [2 marks]

A cylindrical container with a radius of 8 cm is placed on a flat surface. The container is filled with water to a height of 12 cm, as shown in the following diagram.



**diagram not to scale**

Find the volume of water in the container.

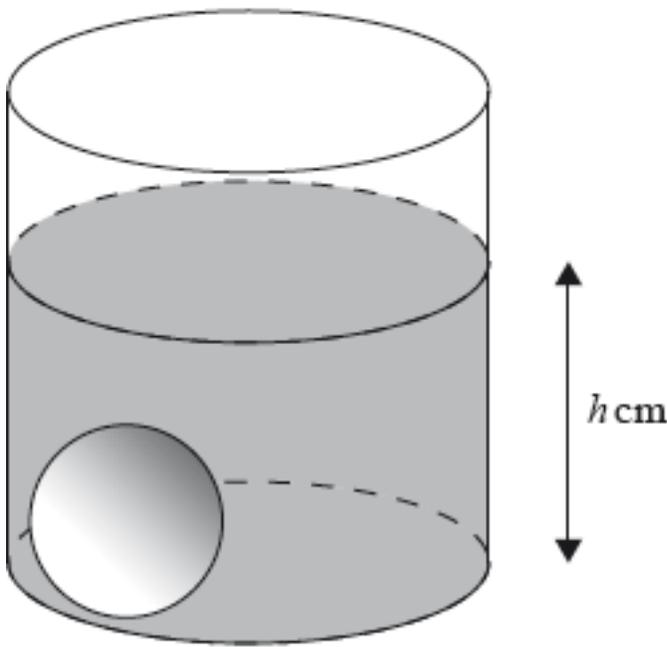
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14b. [4 marks]

A heavy ball with a radius of 2.9 cm is dropped into the container. As a result, the height of the water increases to  $h$  cm, as shown in the following diagram.



**diagram not to scale**

Find the value of  $h$ .

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15a. [3 marks]

Tomás is playing with sticks and he forms the first three diagrams of a pattern. These diagrams are shown below.



Diagram 1



Diagram 2



Diagram 3

Tomás continues forming diagrams following this pattern.

Diagram  $n$  is formed with 52 sticks. Find the value of  $n$ .

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15b. [3 marks]

Tomás forms a total of 24 diagrams.

Find the total number of sticks used by Tomás for all 24 diagrams.

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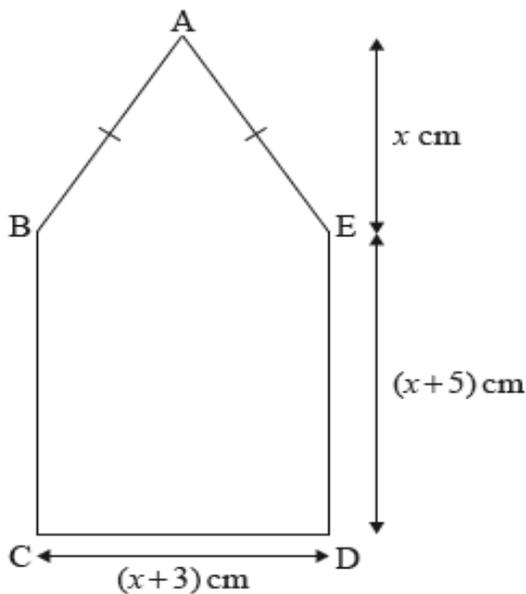
16a. [2 marks]

The base of an electric iron can be modelled as a pentagon ABCDE, where:

BCDE is a rectangle with sides of length  $(x + 3)$  cm and  $(x + 5)$  cm;

ABE is an isosceles triangle, with  $AB = AE$  and a height of  $x$  cm;

the area of ABCDE is  $222 \text{ cm}^2$ .



Write down an **equation** for the area of ABCDE using the above information.

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**16b.** [2 marks]

Show that the equation in part (a)(i) simplifies to  $3x^2 + 19x - 414 = 0$ .

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**16c.** [2 marks]

Find the length of CD.

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**16d.** [3 marks]

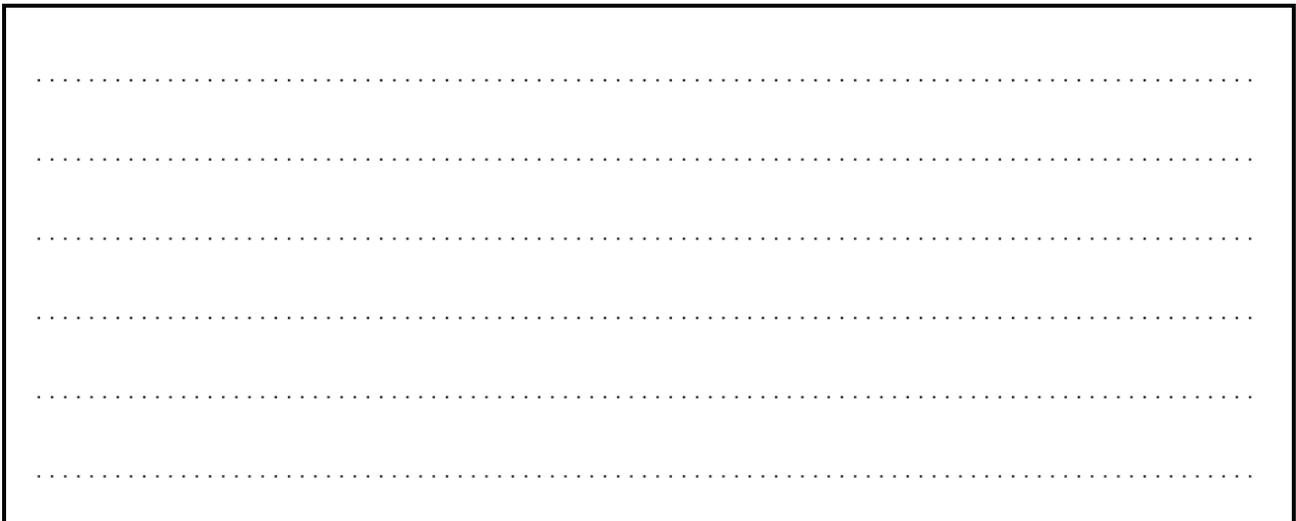
Show that angle  $\hat{BAE} = 67.4^\circ$ , correct to one decimal place.



**16e.** [3 marks]

Insulation tape is wrapped around the perimeter of the base of the iron, ABCDE.

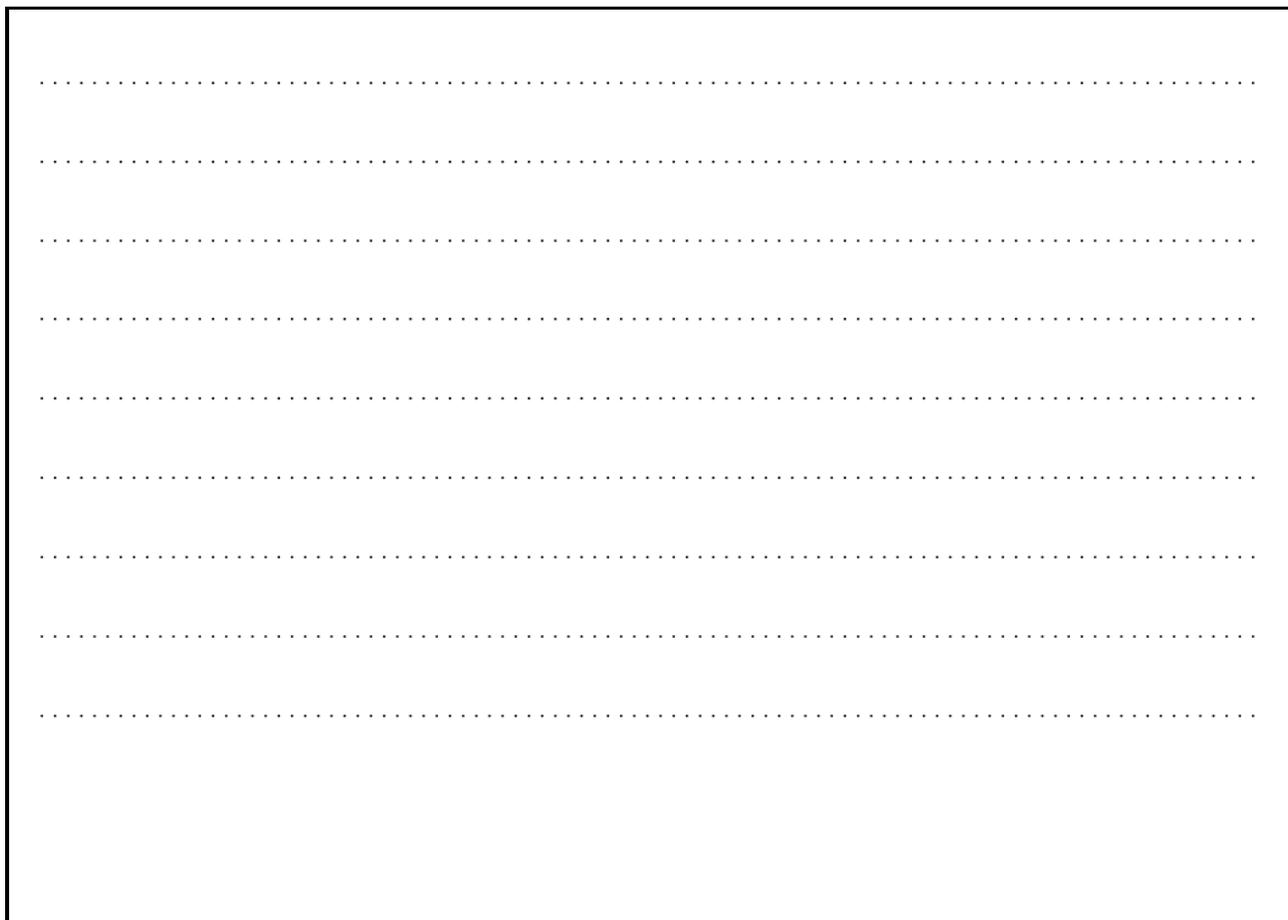
Find the length of the perimeter of ABCDE.



16f. [4 marks]

F is the point on AB such that  $BF = 8 \text{ cm}$ . A heating element in the iron runs in a straight line, from C to F.

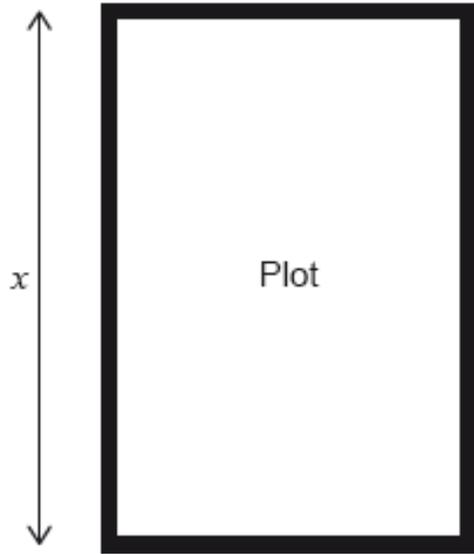
Calculate the length of CF.



## Week 6:

17a. [1 mark]

Violeta plans to grow flowers in a rectangular plot. She places a fence to mark out the perimeter of the plot and uses 200 metres of fence. The length of the plot is  $x$  metres.



Show that the width of the plot, in metres, is given by  $100 - x$ .

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17b. [1 mark]

Write down the area of the plot in terms of  $x$ .

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17c. [2 marks]

Violeta places the fence so that the area of the plot is maximized.

Find the value of  $x$  that maximizes the area of the plot.

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17d. [2 marks]

By selling her flowers, Violeta earns 2 Bulgarian Levs (BGN) per square metre of the plot.

Show that Violeta earns 5000 BGN from selling the flowers grown on the plot.

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17e. [3 marks]

Violeta wants to invest her 5000 BGN.

A bank offers a nominal annual interest rate of 4%, compounded **half-yearly**.

Find the amount of money that Violeta would have after 6 years. Give your answer correct to two decimal places.

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17f. [3 marks]

Find how long it would take for the interest earned to be 2000 BGN.

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17g. [2 marks]

Another bank offers an interest rate of  $r\%$  compounded **annually**, that would allow her to double her money in 12 years.

Find the lowest possible value for  $r$ .

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**18a.** [2 marks]

Let 
$$p = \frac{\cos x + \sin y}{\sqrt{w^2 - z}}$$
,

where  $x = 36^\circ$ ,  $y = 18^\circ$ ,  $w = 29$  and  $z = 21.8$ .

Calculate the value of  $p$ . Write down your full calculator display.

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**18b.** [2 marks]

Write your answer to part (a)

(i) correct to two decimal places;

(ii) correct to three significant figures.

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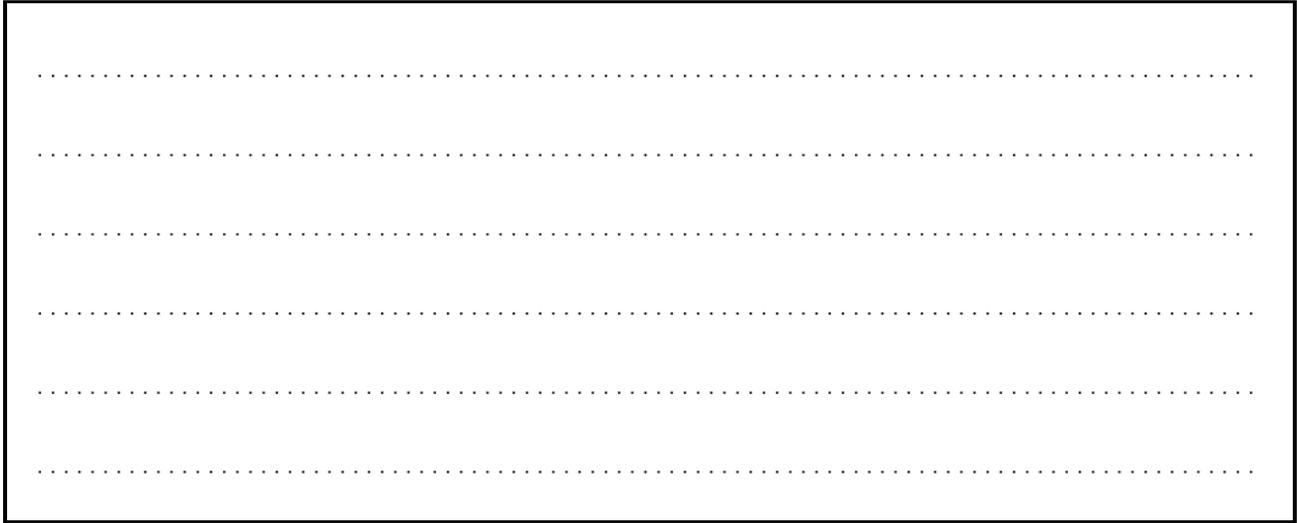
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18c. [2 marks]

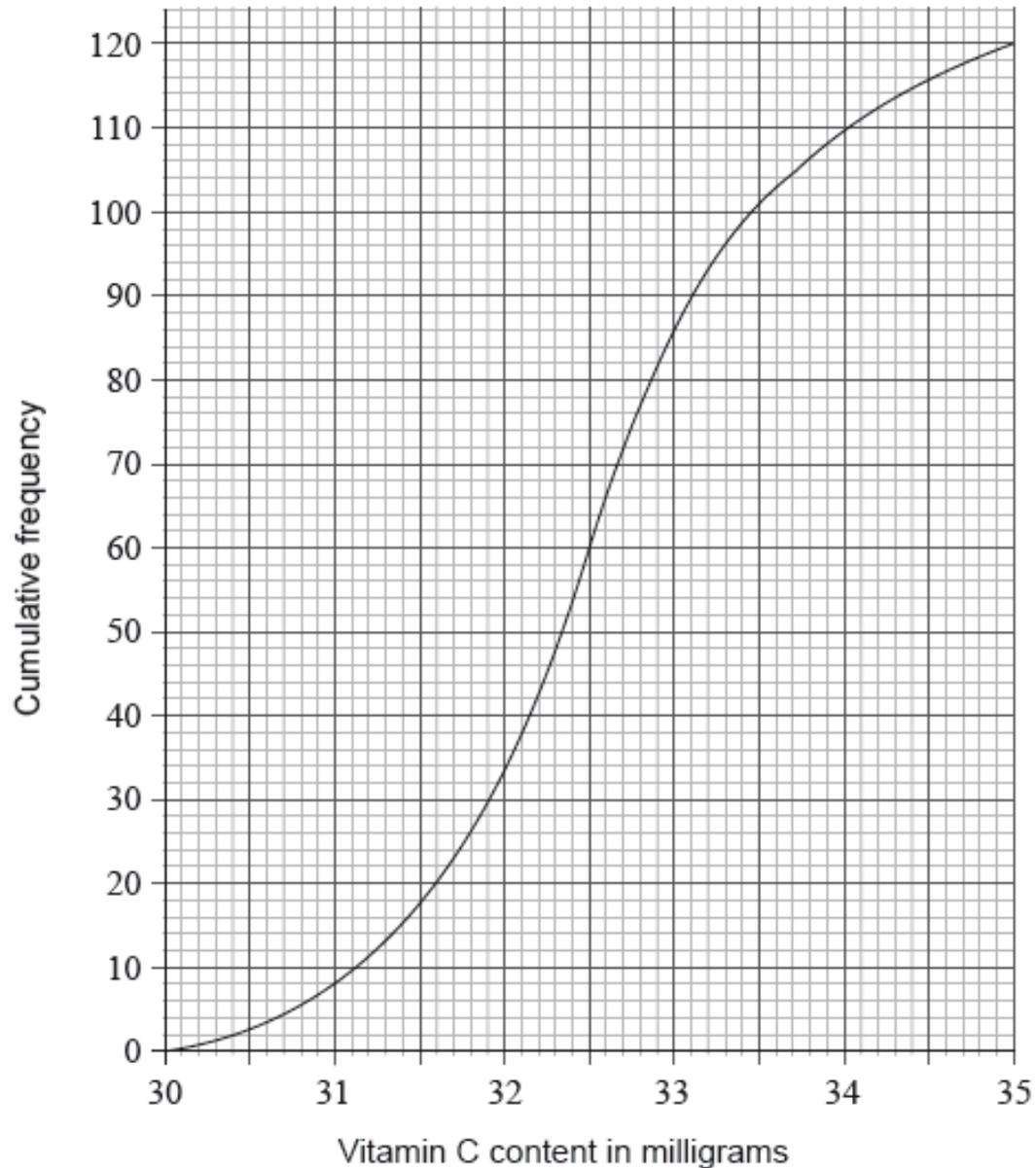
Write your answer to **part (b)(ii)** in the form  $a \times 10^k$ , where  $1 \leq a < 10$ ,  $k \in \mathbb{Z}$ .



## Week 7:

19a. [3 marks]

A sample of 120 oranges was tested for Vitamin C content. The cumulative frequency curve below represents the Vitamin C content, in milligrams, of these oranges.



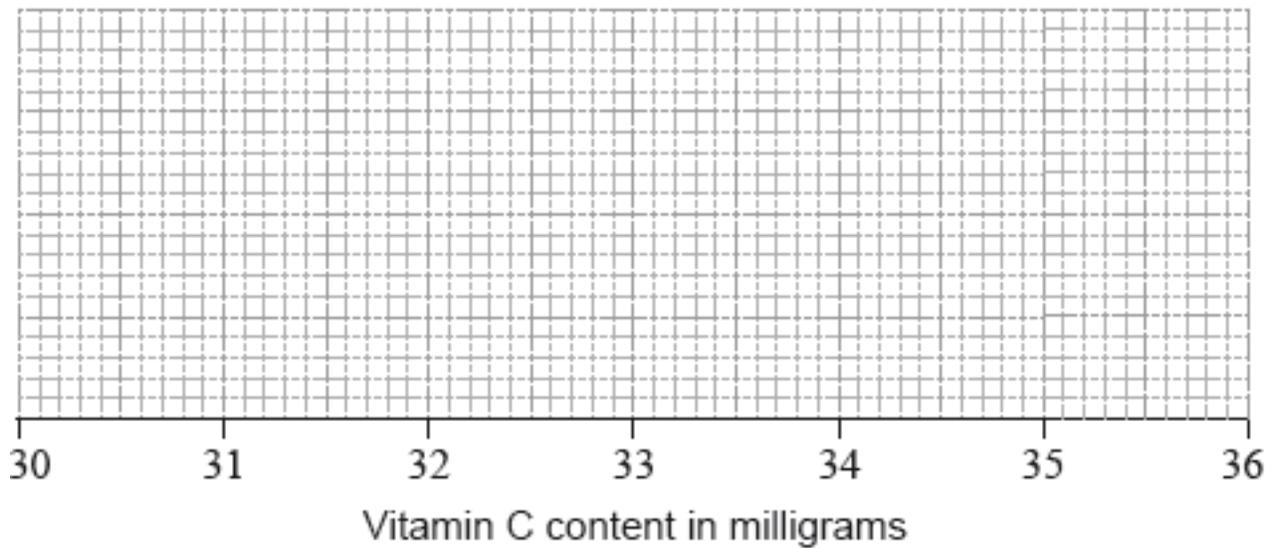
Giving your answer to one decimal place, write down the value of

- (i) the median level of Vitamin C content of the oranges in the sample;
- (ii) the lower quartile;
- (iii) the upper quartile.

**19b.** [3 marks]

The minimum level of Vitamin C content of an orange in the sample was 30.1 milligrams. The maximum level of Vitamin C content of an orange in the sample was 35.0 milligrams.

Draw a box-and-whisker diagram on the grid below to represent the Vitamin C content, in milligrams, for this sample.



**20a.** [1 mark]

Passengers of Flyaway Airlines can purchase tickets for either Business Class or Economy Class.

On one particular flight there were 154 passengers.

Let  $x$  be the number of Business Class passengers and  $y$  be the number of Economy Class passengers on this flight.

Use the above information to write down an equation in  $x$  and  $y$ .

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**20b.** [1 mark]

On this flight, the cost of a ticket for each Business Class passenger was 320 euros and the cost of a ticket for each Economy Class passenger was 85 euros. The total amount that Flyaway Airlines received for these tickets was **14 970 euros**.

Use the information about the cost of tickets to write down a second equation in  $x$  and  $y$ .

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**20c.** [2 marks]

Find the value of  $x$  and the value of  $y$ .

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**20d.** [2 marks]

The airline's finance officer wrote down the total amount received by the airline for these tickets as 14 270 euros.

Find the percentage error.

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**21a.** [2 marks]

The golden ratio,  $r$ , was considered by the Ancient Greeks to be the perfect ratio between the lengths of two adjacent sides of a rectangle. The exact value of  $r$  is  $\frac{1+\sqrt{5}}{2}$ .

Write down the value of  $r$

- i) correct to 5 significant figures;
- ii) correct to 2 decimal places.

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**21b.** [1 mark]

Phidias is designing rectangular windows with adjacent sides of length  $x$  metres and  $y$  metres. The area of each window is  $1 \text{ m}^2$ .

Write down an equation to describe this information.

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**21c.** [1 mark]

Phidias designs the windows so that the ratio between the longer side,  $y$ , and the shorter side,  $x$ , is the golden ratio,  $r$ .

Write down an equation in  $y$ ,  $x$  and  $r$  to describe this information.

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**21d.** [2 marks]

Find the value of  $x$ .

A large rectangular box with a solid black border, containing six horizontal dotted lines for writing.

## Week 8:

22a. [2 marks]

One of the locations in the 2016 Olympic Games is an amphitheatre. The number of seats in the first row of the amphitheatre,  $u_1$ , is 240. The number of seats in each subsequent row forms an arithmetic sequence. The number of seats in the sixth row,  $u_6$ , is 270.

Calculate the value of the common difference,  $d$ .

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22b. [2 marks]

There are 20 rows in the amphitheatre.

Find the **total** number of seats in the amphitheatre.

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22c. [2 marks]

Anisha visits the amphitheatre. She estimates that the amphitheatre has 6500 seats.

Calculate the percentage error in Anisha's estimate.

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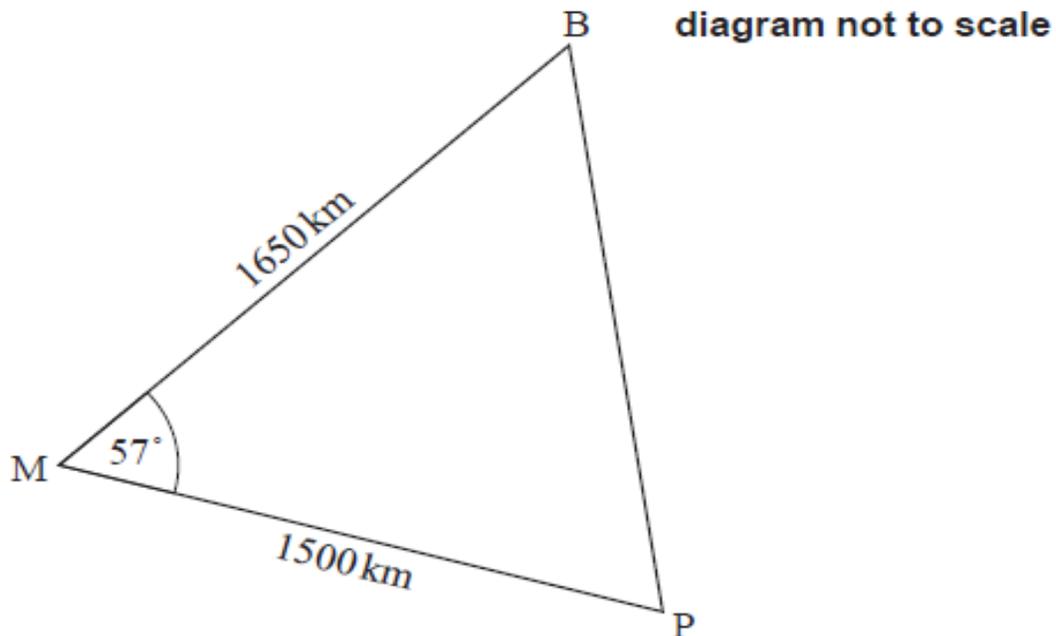
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23a. [3 marks]

When Bermuda (B), Puerto Rico (P), and Miami (M) are joined on a map using straight lines, a triangle is formed. This triangle is known as the Bermuda triangle.

According to the map, the distance MB is 1650 km, the distance MP is 1500 km and angle BMP is  $57^\circ$ .



Calculate the distance from Bermuda to Puerto Rico, **BP**.

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**23b.** [3 marks]

Calculate the area of the Bermuda triangle.

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24a. [2 marks]

Prachi is on vacation in the United States. She is visiting the Grand Canyon.

When she reaches the top, she drops a coin down a cliff. The coin falls down a distance of 5 metres during the first second, 15 metres during the next second, 25 metres during the third second and continues in this way. The distances that the coin falls during each second forms an arithmetic sequence.

(i) Write down the common difference,  $d$ , of this arithmetic sequence.

(ii) Write down the distance the coin falls during the fourth second.

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24b. [2 marks]

Calculate the distance the coin falls during the 15<sup>th</sup> second.

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24c. [3 marks]

Calculate the **total** distance the coin falls in the first **15** seconds. Give your answer in kilometres.

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24d. [3 marks]

Prachi drops the coin from a height of **1800** metres above the ground.

Calculate the time, to the nearest second, the coin will take to reach the ground.

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**24e.** [2 marks]

Prachi visits a tourist centre nearby. It opened at the start of **2015** and in the first year there were **17 000** visitors. The number of people who visit the tourist centre is expected to increase by **10 %** each year.

Calculate the number of people expected to visit the tourist centre in **2016**.

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24f. [3 marks]

Calculate the total number of people expected to visit the tourist centre during the first 10 years since it opened.

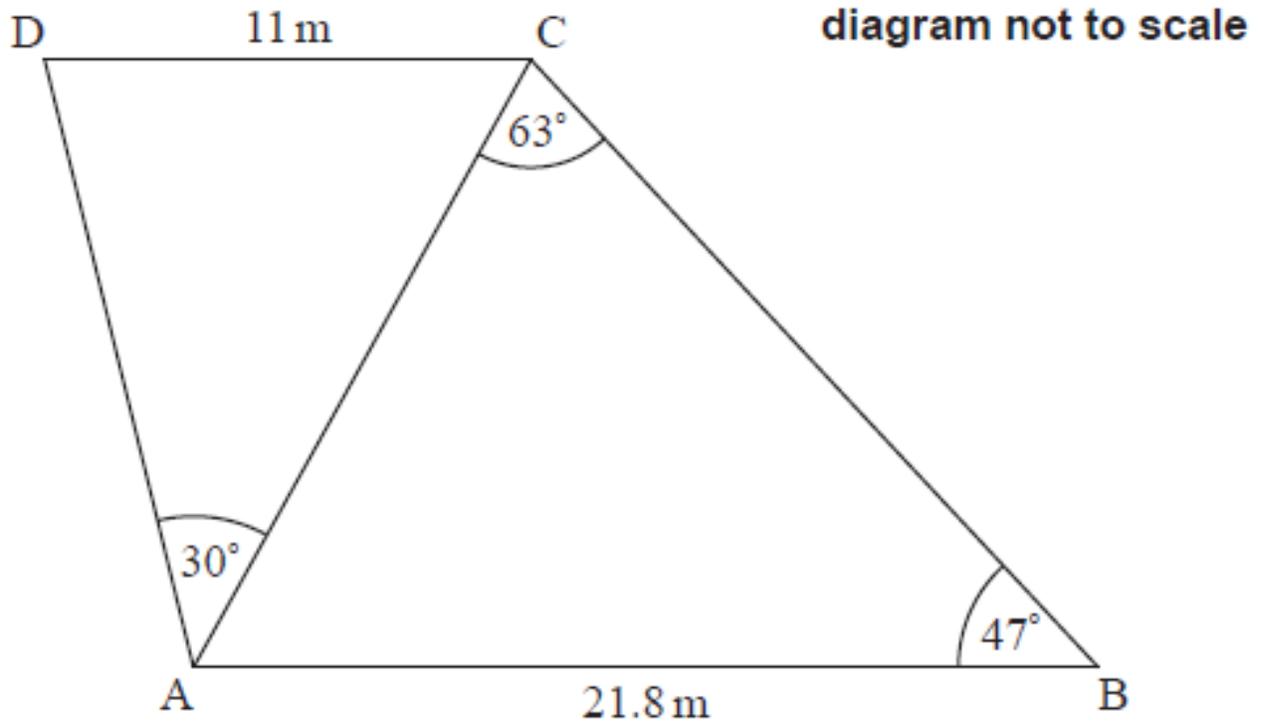


A large rectangular box with a solid black border, containing ten horizontal dotted lines for writing.

## Week 9:

25a. [3 marks]

A playground, when viewed from above, is shaped like a quadrilateral,  $ABCD$ , where  $AB = 21.8$  m and  $CD = 11$  m. Three of the internal angles have been measured and angle  $ABC = 47^\circ$ , angle  $ACB = 63^\circ$  and angle  $CAD = 30^\circ$ . This information is represented in the following diagram.



Calculate the distance  $AC$ .

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**25b.** [3 marks]

Calculate angle **ADC**.

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**25c.** [2 marks]

There is a tree at **C**, perpendicular to the ground. The angle of elevation to the top of the tree from **D** is  $35^\circ$ .

Calculate the height of the tree.

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**25d.** [2 marks]

Chavi estimates that the height of the tree is **6 m**.

Calculate the percentage error in Chavi's estimate.

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25e. [3 marks]

Chavi is celebrating her birthday with her friends on the playground. Her mother brings a 2 litre bottle of orange juice to share among them. She also brings **cone-shaped** paper cups.

Each cup has a vertical height of 10 cm and the top of the cup has a diameter of 6 cm.

Calculate the volume of one paper cup.



**25f.** [3 marks]

Calculate the maximum number of cups that can be completely filled with the **2 litre** bottle of orange juice.



## Week 10:

26a. [2 marks]

$$T = \frac{(\tan(2z)+1)(2 \cos(z)-1)}{y^2-x^2}, \text{ where } x = 9, y = 41 \text{ and } z = 30^\circ.$$

Calculate the **exact** value of  $T$ .

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26b. [2 marks]

Give your answer to  $T$  correct to

(i) two significant figures;

(ii) three decimal places.

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26c. [2 marks]

Pyotr estimates the value of  $T$  to be 0.002.

Calculate the percentage error in Pyotr's estimate.

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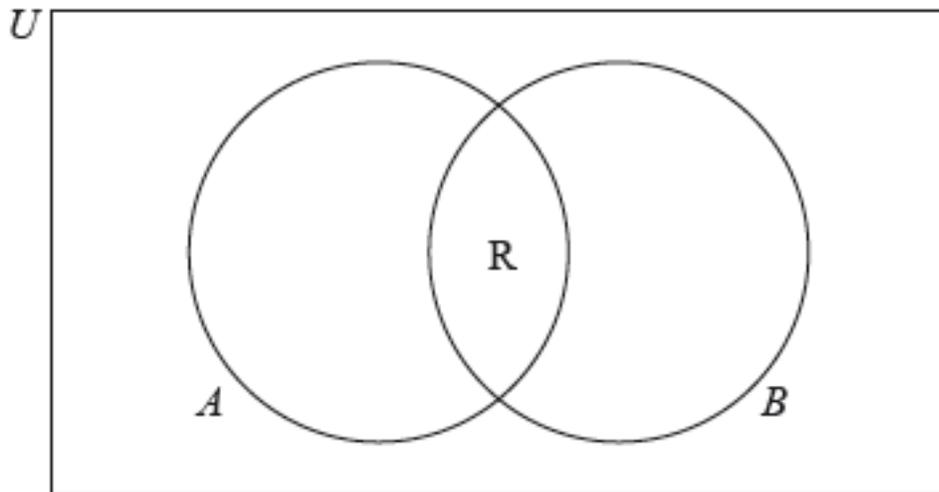
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27a. [3 marks]

Tuti has the following polygons to classify: rectangle (R), rhombus (H), isosceles triangle (I), regular pentagon (P), and scalene triangle (T).

In the Venn diagram below, set  $A$  consists of the polygons that have at least one pair of parallel sides, and set  $B$  consists of the polygons that have at least one pair of equal sides.



Complete the Venn diagram by placing the letter corresponding to each polygon in the appropriate region. For example, R has already been placed, and represents the rectangle.

.....

.....

.....

**27b.** [3 marks]

State which polygons from Tuti's list are elements of

(i)  $A \cap B$ ;

(ii)  $(A \cup B)'$ .

.....

.....

.....

**28a.** [1 mark]

Ludmila takes a loan of 320 000 Brazilian Real (BRL) from a bank for two years at a nominal annual interest rate of 10%, **compounded half yearly**.

Write down the number of times interest is added to the loan in the two years.

28b. [3 marks]

Calculate the **exact** amount of money that Ludmila must repay at the end of the two years.

28c. [2 marks]

Ludmila estimates that she will have to repay 360 000 BRL at the end of the two years.

Calculate the percentage error in her estimate.

29. [6 marks]

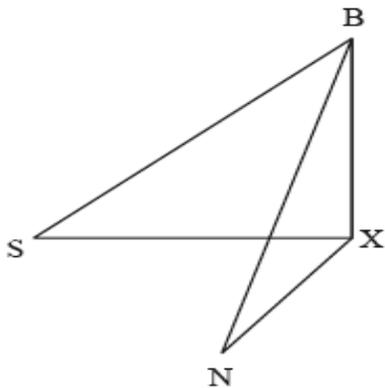
Barry is at the top of a cliff, standing 80 m above sea level, and observes two yachts in the sea.

“Seaview” ( $S$ ) is at an angle of depression of  $25^\circ$ .

“Nauti Buoy” ( $N$ ) is at an angle of depression of  $35^\circ$ .

The following three dimensional diagram shows Barry and the two yachts at  $S$  and  $N$ .

$X$  lies at the foot of the cliff and angle  $\mathbf{SXN} = 70^\circ$ .

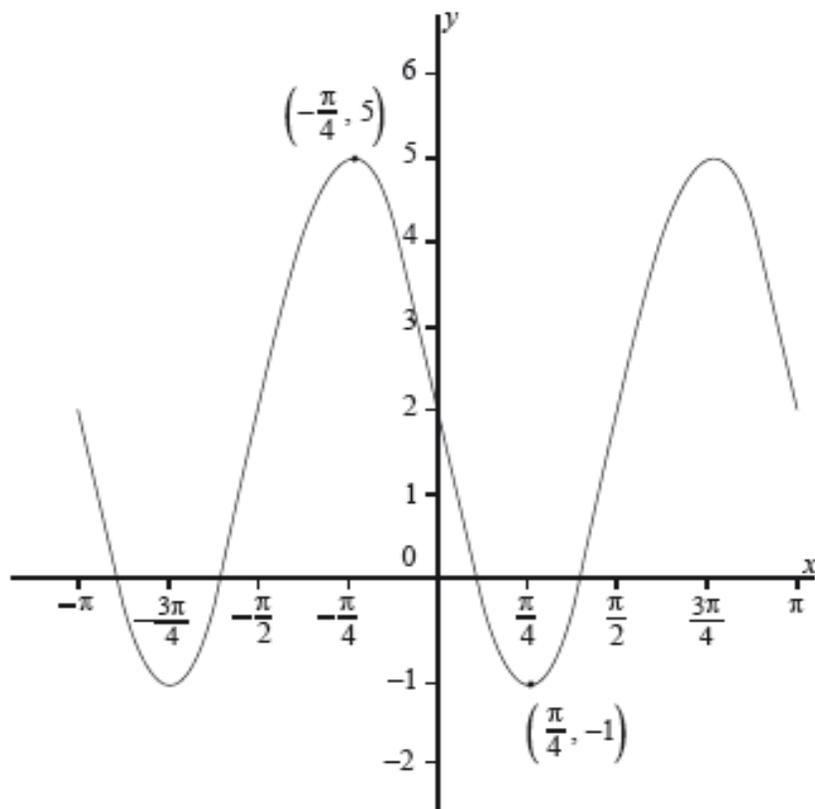


Find, to 3 significant figures, the distance between the two yachts.

A large rectangular box with a solid black border, containing 15 horizontal dotted lines for writing the solution.

30a. [4 marks]

A function is defined by  $f(x) = A \sin(Bx) + C$ ,  $-\pi \leq x \leq \pi$ , where  $A, B, C \in \mathbb{Z}$ . The following diagram represents the graph of  $y = f(x)$ .



Find the value of

(i)  $A$ ;

(ii)  $B$ ;

(iii)  $C$ .

.....
.....
.....

30b. [2 marks]

Solve  $f(x) = 3$  for  $0 \leq x \leq \pi$ .

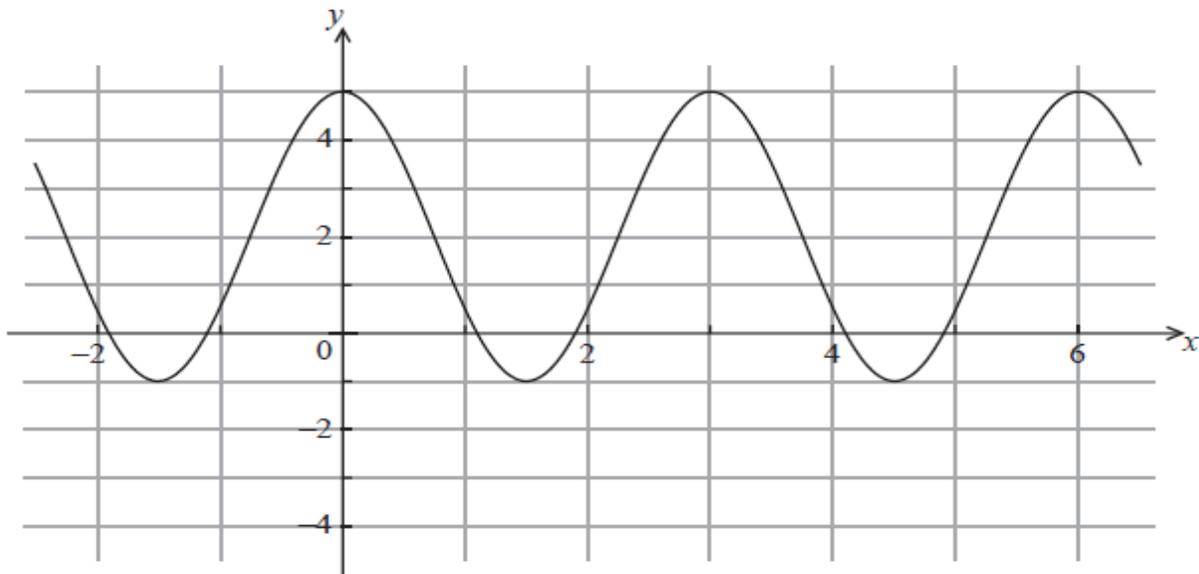
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31. [4 marks]

The graph below shows  $y = a \cos(bx) + c$ .



Find the value of  $a$ , the value of  $b$  and the value of  $c$ .

32. [6 marks]

Consider the triangle ABC where  $\hat{BAC} = 70^\circ$ ,  $AB = 8$  cm and  $AC = 7$  cm. The point D on the side BC is such that  $\frac{BD}{DC} = 2$ .

Determine the length of AD.

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